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M12H Section 4.6A Solving Trigonometric Equations with Identities:

1. Given each equation below, indicate which of the following are Identities and which ones are not. State the ones that are false::

a) $\sin(a+b)=\sin a+\sin b$ ✗	b) $\cos(-x)=\cos(x)$ ✓	c) $\sin(-x)=-\sin(x)$ ✓
d) $\sin 2x=2\sin x$ ✗	e) $\frac{\sin 2\theta}{2}=\sin\theta$ ✗	f) $\sin 2\theta=2\sin\theta\cos\theta$ ✓
g) $\tan\theta=\frac{\cos\theta}{\sin\theta}$ ✗	h) $\tan\theta=\frac{\sin\theta}{\cos\theta}$ ✓	i) $\csc\theta=\frac{1}{\cos\theta}$ ✗
j) $\sin^2\theta+\cos^2\theta=1$ ✓	k) $\cos^{-1}(\theta)=\frac{1}{\cos\theta}$ ✗	L) $\sec\theta=\frac{1}{\cos\theta}$ ✓
M) $\csc(-\theta)=-\frac{1}{\sin\theta}$ ✓	n) $\sin(a+b)=\sin a\cos b+\sin b\cos a$ ✓	0) $\tan\theta=\frac{\sin\theta}{\cos\theta}$ ✓
p) $\sin 2\theta=1\rightarrow\sin\theta=\frac{1}{2}$ ✗	q) $\sin(a-b)=-\sin(b-a)$ ✓	r) $\sin(\pi-\theta)=\sin\theta$ ✓
s) $\cos 2\theta=1-2\sin^2\theta$ $\cos 2\theta = \cos^2\theta-\sin^2\theta$ $1-\sin^2\theta-\sin^2\theta$ ✓	t) $\cos 4\theta=1-2\sin^2 2\theta$ ✓	u) $\sin\theta=\cos\left(\frac{\pi}{2}-\theta\right)$ ✓
v) $\sin(-\theta)+d=-\sin\theta+d$ ✓	w) $\cos(a-b)=\cos a\cos b+\sin a\sin b$ ✓	x) $\cos(a-b)=\cos a-\cos b$ ✗
y) $\sec(-\theta)=\sec\theta$ $\frac{1}{\cos\theta}=\cos\theta$ ✓	z) $\tan\left(\frac{a}{b}\right)=\frac{\tan a}{\tan b}$ ✗	aa) $\tan(a-b)=\frac{\sin(a-b)}{\cos(a-b)}$ ✓
Ab) $\cos^{-1}\theta=\frac{adj}{hyp}$ ✗ side angle	Ac) $\cos^{-1}\left(\frac{adj}{hyp}\right)=\theta$ ✓	Ad) $1+\cot^2\theta=\csc^2\theta$ $\sin^2\theta+\cos^2\theta=1$ ✓

2. Solve the following equations for $0 \le \theta \le 2\pi$. Use the trigonometric Identities shown in class. Make sure to check for extraneous roots: Show all your work and steps:

i) $\cos 2\theta = \cos^2\theta$

$\cancel{\cos^2\theta} - \sin^2\theta = \cancel{\cos^2\theta}$

$\sin^2\theta = 0$

$\boxed{\theta = 0, \pi, 2\pi}$

ii) $\cos\theta \sin\theta = \dfrac{1}{4}$

$\dfrac{\sin 2\theta}{2} = \dfrac{1}{4}$

$\sin 2\theta = \dfrac{1}{2}$

$2\theta = \dfrac{\pi}{6}$

$\boxed{\theta = \dfrac{\pi}{12}, \dfrac{13\pi}{12},}$

iii) $\sin 2\theta = 2\cos\theta$

$2\sin\theta \cancel{\cos\theta} - 2\cancel{\cos\theta}$

$\sin\theta = 1$

$\boxed{\theta = \dfrac{\pi}{2}}$

iv) $\cos\theta \cot 2\theta = \cos\theta$

$\cos\theta(\cot 2\theta - 1) = 0$

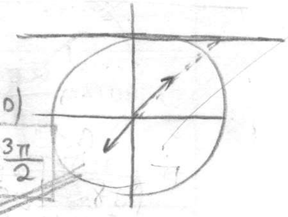
$\cot 2\theta = 1$ $\quad$ $\cos\theta = 0$

$2\theta = \cot^{-1}(1) = \dfrac{\pi}{4},$

$\boxed{\theta = \dfrac{\pi}{8}, \dfrac{5\pi}{8}, \dfrac{9\pi}{8}, \dfrac{13\pi}{8}}$

$\theta = \cos^{-1}(0)$

$\boxed{\theta = \dfrac{\pi}{2}, \dfrac{3\pi}{2}}$



v) $\cos\theta \tan\theta = \cos\theta$

$\cos\theta(\tan\theta - 1) = 0$

$\cos\theta = 0$ $\quad$ $\tan\theta = 1$

$\theta = \cos^{-1}(0)$ $\quad$ $\theta = \tan^{-1}(1)$

$\boxed{\theta = \dfrac{\pi}{2}, \dfrac{3\pi}{2}}$ $\quad$ $\boxed{\theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}}$

vi) $\cot\theta = 2\sin 2\theta$

$\dfrac{\cos\theta}{\sin\theta} = 4\sin\theta\cos\theta$

$\cos\theta = 4\sin^2\theta\cos\theta$

$\cos\theta(1 - 4\sin^2\theta) = 0$

$\cos\theta(1 - 2\sin\theta)(1 + 2\sin\theta) = 0$

$\cos\theta = 0$

$\theta_1 = \cos^{-1}(0) = \boxed{\dfrac{\pi}{2}, \dfrac{3\pi}{2}}$

$2\sin\theta = 1$

$\theta = \sin^{-1}\left(\dfrac{1}{2}\right) = \boxed{\dfrac{\pi}{6}, \dfrac{5\pi}{6}}$

$2\sin\theta = -1$

$\sin\theta = -\dfrac{1}{2}$

$\theta = \sin^{-1}\left(-\dfrac{1}{2}\right) = \boxed{\dfrac{7}{6}\pi, \dfrac{11\pi}{6}}$

PLUG ALL IN!

Vii) $\cos 2\theta - \cos\theta = 0$

$\cos^2\theta - \sin^2\theta - \cos\theta = 0$

$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$

$2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\theta_1 = \cos^{-1}\left(-\dfrac{1}{2}\right) = \boxed{\dfrac{2\pi}{3}, \dfrac{4\pi}{3}}$ $\quad$ $\theta_2 = \cos^{-1}(1) = \boxed{0, 2\pi}$

Viii) $\cos(\theta + \pi) - \cos^2\theta = 0$

$-\cos\theta - \cos^2\theta = 0$

$\cos\theta(1 + \cos\theta) = 0$

$\cos\theta = 0$ $\quad$ $1 + \cos\theta = 0$

$\theta = \cos^{-1}(0) = \boxed{\dfrac{\pi}{2}, \dfrac{3\pi}{2}}$ $\quad$ $\cos\theta = -1$

$\theta = \cos^{-1}(-1)$

$\boxed{\theta = \pi}$

ix) $\sin^2\theta + 2\cos^2\theta = \dfrac{7}{4}$

$= \underbrace{\sin^2\theta + \cos^2\theta}_{=1} + \cos^2\theta = \dfrac{7}{4}$

$\cos^2\theta = \dfrac{3}{4}$

$\cos\theta = \pm\dfrac{\sqrt{3}}{2}$

$\theta = \cos^{-1}\left(\pm\dfrac{\sqrt{3}}{2}\right)$

$\boxed{\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}}$

x) $3 - 3\csc\theta + \cot^2\theta = 0$

$3 - \dfrac{3}{\sin\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} = 0$

$3 - \dfrac{3\sin\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} = 0$

$3 + \dfrac{1 - \sin^2\theta - 3\sin\theta}{\sin^2\theta} = 0$

$1 - \sin^2\theta - 3\sin\theta = -3\sin^2\theta$

$2\sin^2\theta - 3\sin\theta + 1 = 0$

$(\sin\theta - 1)(2\sin\theta - 1) = 0$

$\sin\theta = 1$

$\theta = \sin^{-1}(1)$

$\boxed{\theta = \dfrac{\pi}{2}}$

$\sin\theta = \dfrac{1}{2}$

$\theta = \sin^{-1}\left(\dfrac{1}{2}\right)$

$\boxed{\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}}$

xi) $2\sin^2\theta - \cos\theta - 1 = 0$

$2(1-\cos^2\theta) - \cos\theta - 1 = 0$

$-2\cos^2\theta - \cos\theta + 1 = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{3}, \frac{5\pi}{3}}$

$\cos\theta = -1$

$\theta = \cos^{-1}(-1)$

$= \boxed{\pi}$

xiii) $3\cos^2\theta - 8\cos\theta + 4 = 0$

$(3\cos\theta - 2)(\cos\theta - 2) = 0$

$\cos\theta = \frac{2}{3}$ $\qquad$ $\cancel{\cos\theta = 2}$ rej

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$

$\boxed{\theta = 0.84, 5.44}$

sii) $\cot\theta = \sin 3\theta$

$\frac{\cos\theta}{\sin\theta} = 3\sin\theta - 4\sin^3\theta$

$\cos\theta = 3\sin^2\theta - 4\sin^4\theta$

$\cos\theta = 3(1-\cos^2\theta) - 4(1-\cos^2\theta)^2$

$\cos\theta = 3 - 3\cos^2\theta - 4 + 8\cos^2\theta - 4\cos^4\theta$

$-4\cos^4\theta + 5\cos^2\theta - \cos\theta - 1 = 0$

$4\cos^4\theta - 5\cos^2\theta + \cos\theta + 1 = 0$?

xii) $2\sin^2\theta\cot\theta - 5\sin\theta\cot\theta - 3\cot\theta = 0$

$\cot\theta(2\sin^2\theta - 5\sin\theta - 3) = 0$

$\cot\theta = 0$

$\theta = \cot^{-1}(0)$

$\boxed{\theta = \frac{\pi}{2}, \frac{3\pi}{2}}$

$2\sin^2\theta - 5\sin\theta - 3 = 0$

$(\sin\theta - 3)(2\sin\theta + 1) = 0$

$\cancel{\sin\theta = 3}$ rej

$\sin\theta = -\frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{7\pi}{6}, \frac{11\pi}{6}}$

si) $6\sin\theta(\tan\theta) - \tan\theta - \sec\theta = 0$

$\frac{6\sin^2\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta} = 0$ $\quad$ $\cos\theta \neq 0$

$6\sin^2\theta - \sin\theta - 1 = 0$

$(3\sin\theta + 1)(2\sin\theta - 1) = 0$

$\sin\theta = -\frac{1}{3}$

$\theta = \sin^{-1}\left(-\frac{1}{3}\right)$

$\boxed{\theta = 5.94, 3.48}$ NO NPV

$2\sin\theta = 1 \Rightarrow \theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6}}$

siii) $8\cot\theta\cos\theta - 6\sqrt{3}\cot\theta + 3\csc\theta = 0$

$\frac{8\cos^2\theta}{\sin\theta} - \frac{6\sqrt{3}\cos\theta}{\sin\theta} + \frac{3}{\sin\theta} = 0$ $\quad$ $\sin\theta \neq 0$

$8\cos^2\theta - 6\sqrt{3}\cos\theta + 3 = 0$

$\cos\theta = \frac{6\sqrt{3} \pm \sqrt{(6\sqrt{3})^2 - 4(8)(3)}}{2(8)} = \frac{6\sqrt{3} \pm 2\sqrt{3}}{2(8)} = \frac{\sqrt{3}(3 \pm 1)}{8}$

$\theta = \cos^{-1}\left(\frac{\sqrt{3}(3\pm1)}{8}\right) = \boxed{1.12, 5.16, 0.52, 5.76}$

3. Find all the NPV(s) for each of the following expressions:

i) $\frac{\sec\theta}{2\sin\theta + 1}$

$2\sin\theta + 1 \neq 0$

$\sin\theta \neq -\frac{1}{2}$

$\boxed{\theta \neq \frac{7\pi}{6}, \frac{11\pi}{6}}$

ii) $\frac{\sin\theta\sec\theta}{\cot\theta}$

$\cos\theta \neq 0$

$\sin\theta \neq 0$

$\boxed{\theta \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi}$

iii) $\frac{\sec^2\theta - 1}{1 + \tan^2\theta}$ $\quad$ $\sec\theta = \frac{1}{\cos\theta}$

$\cos\theta \neq 0 \Rightarrow \theta = \boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$

$1 + \tan^2\theta \neq 0$

iv) $\frac{\cos\theta}{1+\sin\theta} + \frac{\cot\theta}{1-\sin\theta}$

$\sin\theta \neq 0 \Rightarrow \boxed{\theta \neq 0, \pi, 2\pi}$

$1 + \sin\theta \neq 0 \Rightarrow \sin\theta \neq -1 \Rightarrow \boxed{\theta \neq \sin^{-1}(-1) \neq \frac{3\pi}{2}}$

$1 - \sin\theta \neq 0 \Rightarrow \theta \neq \sin^{-1}(1) \Rightarrow \boxed{\theta \neq \frac{\pi}{2}}$

v) $\frac{1 + \cos^2\theta - \sin\theta}{\sin\theta\cos\theta\tan\theta}$

$\sin\theta\cos\theta\tan\theta \neq 0$

$\sin\theta \neq 0$ $\quad$ $\boxed{\theta \neq 0, \pi, 2\pi}$

$\cos\theta \neq 0$ $\quad$ $\boxed{\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}}$

vi) $\frac{\cos 2\theta - 1}{3\sin 2\theta - 2}$

$3\sin 2\theta - 2 \neq 0$

$\sin 2\theta \neq \frac{2}{3}$

$2\theta \neq \sin^{-1}\left(\frac{2}{3}\right) = 0.7297, 2.4119$

$\boxed{\theta = 0.36, 3.87, 1.21, 4.35^R}$

vii) $\cot\theta(\tan\theta)(\csc\theta)\sin\theta$

$\sin\theta \neq 0 \Rightarrow \theta \neq \boxed{0, \pi, 2\pi}$

viii) $\frac{1 - \cot\theta}{1 + \tan\theta} \times \frac{\sin\theta}{\sec\theta}$

$\cos\theta \neq 0 \Rightarrow \theta \neq \boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$

$1 + \tan\theta \neq 0$

$\tan\theta \neq -1$

$\theta \neq \tan^{-1}(-1)$

$\boxed{\theta \neq \frac{3\pi}{4}, \frac{7\pi}{4}}$

typo; no "2" there

4. Prove the following Identities. Also indicate any restrictions on the domain:

i) $\csc\theta\sin 2\theta - \sec\theta\cos 2\theta = \sec\theta$

$= \csc\theta \cdot 2\sin\theta\cos\theta - \sec\theta\cdot(\cos^2\theta - \sin^2\theta)$ double angle identity

$= \frac{1}{\sin\theta}\cdot 2\sin\theta\cos\theta - \frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta}$ Reciprocal identities

$= 2\cos\theta - \cos\theta + \frac{1-\cos^2\theta}{\cos\theta}$ simplify and use pythag identity

$= \frac{1}{\cos\theta} = \boxed{\sec\theta}$ $\boxed{\sin\theta\neq0,\ \cos\theta\neq0}$ = RHS

ii) $\frac{\sin\theta}{1-\sin\theta}+\frac{\sin\theta}{1+\sin\theta} = \cancel{2}\sin 2\theta\sec^3\theta$

$= \frac{\sin\theta(1+\sin\theta)+\sin\theta(1-\sin\theta)}{1-\sin^2\theta}$ combine fractions

$= \frac{2\sin\theta}{\cos^2\theta}$ expand, simplify, change denominator with pythag. identity

$= \frac{2\sin\theta\times\cos\theta}{\cos^2\theta\times\cos\theta}$ multiply both sides by $\cos\theta$

$= \boxed{\sin 2\theta\cdot\sec^3\theta}$ $\sin 2\theta = 2\sin\theta\cos\theta$, $\frac{1}{\cos^3\theta} = \sec^3\theta$ $\boxed{\sin\theta\neq\pm1,\ \cos\theta\neq0}$

iii) $\frac{\tan\theta(\cos\theta+\cot\theta)}{\sec\theta+\tan\theta} = \frac{\sin\theta\sin 2\theta}{2-2\cos^2\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}\left(\cos\theta+\frac{\cos\theta}{\sin\theta}\right)}{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}$ make everything in terms of sin & cos.

$= \frac{\sin\theta+1}{\frac{1+\sin\theta}{\cos\theta}} = \cos\theta = \frac{\cos\theta(2-2\cos^2\theta)}{2-2\cos^2\theta}$ multiply both sides by $(2-2\cos^2\theta)$

$\frac{2\cos\theta(\sin^2\theta)}{2-2\cos^2\theta}$ $\boxed{\frac{\sin\theta\sin 2\theta}{2-2\cos^2\theta}}$ = RHS Pythag. identity, double angle identity

$\boxed{\cos\theta,\sin\theta\neq0,\ \sin\theta\neq1}$

iv) $\frac{\cos\theta+\sin\theta\tan\theta}{\sin\theta\sec\theta} = \csc\theta$

$\boxed{\text{NPV: } \sin\theta,\cos\theta\neq0}$

$= \frac{\cos\theta+\frac{\sin^2\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$

$= \frac{\frac{\cos^2\theta}{\cos\theta}+\frac{\sin^2\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$ common denominator and combine

$= \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \csc\theta$ = RHS

v) $\frac{\tan\theta}{\sec\theta+1} = \frac{2\cos\theta-2\cos^2\theta}{\sin 2\theta}$

$\boxed{\text{NPV: } \sin 2\theta\neq0,\ \cos\theta\neq0,\ \sec\theta\neq-1}$

$= \frac{\cancel{2\cos\theta}(1-\cos\theta)}{\cancel{2}\sin\theta\cancel{\cos\theta}}$

multiply top & bottom by $1+\cos\theta$ $= \frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)}$

pythag. identity $= \frac{\sin^2\theta}{\sin\theta(1+\cos\theta)} = \frac{\sin\theta\times\frac{1}{\cos\theta}}{1+\cos\theta\times\frac{1}{\cos\theta}} = \frac{\tan\theta}{\sec\theta+1}$ = LHS

multiply top & bottom by $\frac{1}{\cos\theta}$

vi) $\frac{\sin 2\theta+\sin\theta}{1+\cos\theta+\cos 2\theta} = \cot\left(\frac{\pi}{2}-\theta\right)$

$= \frac{2\sin\theta\cos\theta+\sin\theta}{1+\cos\theta+\cos^2\theta-\sin^2\theta}$

$= \frac{2\sin\theta\cos\theta+\sin\theta}{\cancel{1}+\cos\theta+2\cos^2\theta-\cancel{1}}$ pythag. identity

$= \frac{\sin\theta(\cancel{2\cos\theta+1})}{\cos\theta(\cancel{1+2\cos\theta})}$

$= \tan\theta = \cot\left(\frac{\pi}{2}-\theta\right)$ = RHS

$\boxed{\text{NPV: } \cos\theta+\cos 2\theta\neq-1,\ \tan\theta\neq0}$

vii) $(\sec\theta-\cos\theta)(\csc\theta-\sin\theta) = \frac{\tan\theta}{1+\tan^2\theta}$

NPV: $\cos\theta, \sin\theta\neq0$

$\left(\frac{1}{\cos\theta}-\cos\theta\right)\left(\frac{1}{\sin\theta}-\sin\theta\right)$

$= \left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{\sin^2\theta}{\cos\theta}\right)\left(\frac{\cos^2\theta}{\sin\theta}\right)$ pythag. identity

$= \sin\theta\cos\theta\times\frac{\cos\theta}{\cos\theta} = \frac{\sin\theta\cos^2\theta}{\cos\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}} = \frac{\tan\theta}{1+\tan^2\theta}$ = RHS

viii) $\frac{\sin 4\theta-\sin 2\theta}{\cos 4\theta+\cos 2\theta} = \tan\theta$

$= \frac{2\sin 2\theta\cos 2\theta-\sin 2\theta}{\cos^2 2\theta-\sin^2 2\theta+\cos 2\theta}$ double angle identity

$= \frac{\sin 2\theta(2\cos 2\theta-1)}{2\cos^2 2\theta+\cos 2\theta+1} = \frac{\sin 2\theta(\cancel{2\cos 2\theta-1})}{(\cos 2\theta+1)(\cancel{2\cos\theta-1})}$ factor & cancel

$= \frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta+1} = \frac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cos^{\cancel{2}}\theta} = \boxed{\tan\theta}$ RHS Pyth. identity

$\boxed{\text{NPV: } \cos\theta\neq0;\ \cos 4\theta+\cos 2\theta\neq0}$

ix) $\tan 2\theta = \frac{2}{\cot\theta-\tan\theta}$

$= \frac{2\tan\theta}{1-\tan^2\theta}$ double angle identity

$= \frac{2}{\frac{1}{\tan\theta}-\frac{\tan^2\theta}{\tan\theta}}$

$= \frac{2}{\cot\theta-\tan\theta}$ = RHS

$\boxed{\text{NPV: } \tan\theta=1,\ \cot\theta-\tan\theta\neq0}$

x) $\frac{\sec\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos^3\theta}$

pythag. identity $= \frac{1+\sin\theta}{\cos\theta(1-\sin^2\theta)}$

cancel $= \frac{1+\cos\theta}{\cos\theta(1-\sin\theta)\cancel{(1+\sin\theta)}}$

$\frac{1}{\cos\theta} = \sec\theta$ LHS $= \frac{\sec\theta}{1-\sin\theta}$

$\boxed{\text{NPV: } \cos\theta\neq0,\ \sin\theta\neq1}$